LAMB WAVE ANALYSIS FOR NON-DESTRUCTIVE TESTING OF CONCRETE PLATE STRUCTURES

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Abstract

Multimodal Lamb wave dispersion curves are measured and analyzed to obtain elastic stiffness parameters and thickness of concrete plate structures. With a simple and cost effective field procedure and by utilizing the Multichannel Analysis of Surface Waves (MASW) processing technique, the characteristics of the different modes in experimental Lamb wave dispersion curves can be measured. Lamb waves are guided dispersive waves propagating in plate structures. By matching theoretical Lamb wave dispersion curves with experimental dispersion curves, Young's modulus, Poisson's ratio, and the thickness of the tested structure can be evaluated. A theoretical background with dispersion equations is given along with a practical guide to generate theoretical dispersion curves. Since these pure Lamb wave dispersion curves are only dependent on the plate parameters, the frequency and the phase velocity can be normalized with respect to shear wave velocity and the thickness of the plate. This reduces the calculations during the matching procedure, and one only need to rescale the normalized axis of the dispersion curves to match theoretical and experimental dispersion curves. With a sensitivity analysis we give some recommendations on the matching procedure. The proposed analysis scheme is demonstrated using a case study on a concrete bridge support. Available reference data is in good agreement with the evaluated parameters from the presented analysis scheme.

Introduction

Non-destructive testing (NDT) of civil infrastructures is an important part of maintenance, risk analysis and verification of new structures. Seismic wave based testing techniques have the advantage of measuring fundamental elastic properties (i.e. seismic velocities), by affecting a representable volume in a non-destructive manner. Dynamic Young's modulus (E) and Poisson's ratio (ν) can be directly calculated from measured seismic velocities by using fundamentally correct relationships.

Lamb waves (Lamb, 1917) are guided dispersive waves propagating in free plate structures. By matching theoretical multimodal Lamb wave dispersion curves with experimental ones, shear wave velocity (V_S), Poisson's ratio, and thickness of the tested plate structure can be evaluated. Lamb waves are commonly used in ultrasonic NDT applications including material characterization of elastic plates (Rogers, 1995), viscoelastic plates (Dean, 1989), bonding inspection (Wu and Liu, 1999), coating inspection (Lee and Cheng, 2001), defect inspection (Gilchrist, 1999), and thickness measurements of thin films (Pei et al., 1995).

In NDT of infrastructures Lamb-wave-based testing techniques for concrete slabs and pavements were proposed already in the 1940's. Picket (1945) presented a theoretical analysis on the application of Lamb waves for non-destructive testing of concrete slabs. In the following years several publications reported on this approach and showed promising results (Jones, 1955; Jones 1962; Vidale, 1964; Jones and Thrower, 1965). A key issue for civil engineering applications using Lamb wave testing techniques is whether the free plate boundary conditions are fulfilled. Naturally concrete slabs and pavement asphalt layers are in contact with granular base or subgrade/fill material. Several researchers have been investigating this question and studies indicate that Lamb waves are the prevailing type of stress waves generated with their dispersion characteristics little different from the pure (traction-free on both sides) Lamb case. This theory seems valid as far as the stiffness of the underlying layer is a fraction of the stiffness of the plate (Jones and Thrower, 1965; Martincek 1994; Ryden et al., 2002b). In early concrete and pavement applications using Lamb waves the measuring technique, the steady-state surface wave method, was time consuming and the analysis scheme was limited to the fundamental anti-symmetric mode of propagation because the method was not utilizing the multichannel recording and analysis concepts.

In this paper a practical approach for NDT of pavements and concrete structures utilizing multiple modes of Lamb wave dispersion curves is presented. Both the measuring procedure and the analysis scheme is described. A complete description of how to calculate theoretical multimodal Lamb wave dispersion curves is given. The main difference from the original approach presented by Jones (1955) is that measurements can now be made in the level of multimodal nature of Lamb waves in which the dispersion of individual modes are identified through a 2D (time and space) wave field transformation. Also, the entire procedure of testing can be finished within a couple of minutes instead of hours and with significantly cheaper equipment. Both measurements and analysis are made on the same portable computer and the recorded data can be evaluated directly in the field to obtain the final result.

Theoretical Lamb wave dispersion curves

Pure Lamb waves are guided dispersive waves propagating in an elastic isotropic plate with traction free boundaries, Figure 1. Lamb waves are formed by interference of multiple reflections and mode conversion of longitudinal waves (P-waves) and shear waves (S-waves) at the free surfaces of the plate (Viktorov, 1967).

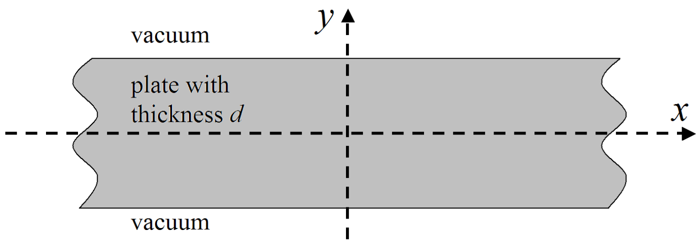


Figure 1. Schematic representation of plate and coordinates.

Lamb waves with particle displacement in both the x- and y-direction actually represents a group of wave types including the bending wave, the Rayleigh wave, and the quasi-longitudinal wave. Lamb (1917) derived the dispersion equation that can handle the transitions between these types of waves. Harmonic wave propagation in the x-direction is only possible for those combinations of frequency (f) and phase velocity (c) corresponding to standing waves in the thickness y-direction. These waves must obey the dispersion equation (Lamb, 1917), from which dispersion curves can be calculated.

$$\frac{\tan \beta \frac{d}{2}}{\tan \alpha \frac{d}{2}} = -\left[\frac{4\alpha\beta k^2}{\left(k^2 - \beta^2\right)^2} \right]^{\pm 1} \qquad \text{Eq. 1}$$

where

$$\alpha^2 = \frac{\omega^2}{V_P^2} - k^2 \qquad \text{Eq. 2}$$

$$\beta^2 = \frac{\omega^2}{V_S^2} - k^2 \qquad \text{Eq. 3}$$

The ± sign on the exponent of the right term of equation 1 represents symmetric (+) and anti-symmetric (-) type of wave propagation with respect to the middle of the plate, Figure 2. The other terms are wave number ($k=\omega/c$) where ω is circular frequency ($\omega=2\pi f$); thickness (d); longitudinal wave velocity (V_P); and shear wave velocity (V_S).

Equation 1 represents the dispersion relation for pure Lamb waves with particle motion in both the x- and the y-direction. Although, the equation was derived long ago and looks quite simple, calculating roots for dispersion curve generation can be challenging, especially in some regions of wave number and frequency (Graff, 1975). Therefore further insight into the practical dispersion curve calculation is given next.

The dispersion relation is a transcendental function and it not straightforward to calculate a k value at any given frequency. A root searching technique has to be used to find the right wave numbers at any given frequency. In general roots are complex, but if only propagating waves are studied the imaginary component can be ignored (Achenbach, 1998). In Figure 3 the absolute value of the anti-symmetric function has been plotted as a function of real wave numbers at frequencies of 1 and 10 kHz. In this example the plate thickness is 0.2 m, V_P=1581 m/s, and V_S=1000 m/s. Roots are visible as minima in the absolute function value.

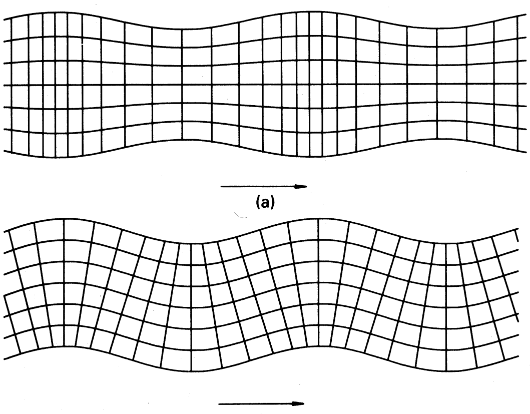


Figure 2. Mode shapes of (a) symmetric and (b) anti-symmetric Lamb wave propagation, from Kuttruff (1991).

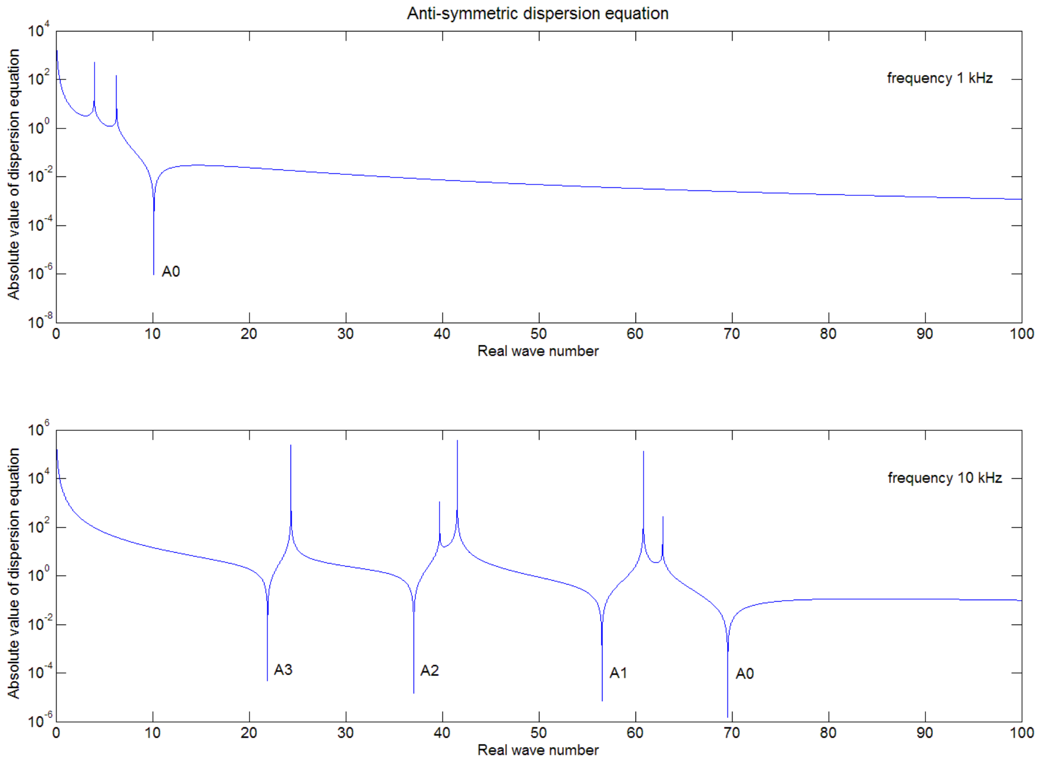


Figure 3. Absolute value of anti-symmetric dispersion equation at 1 kHz (upper) and 10 kHz (lower). Roots of the dispersion equation are visible as minima. The number of roots (at a given frequency) increases with increasing frequency. The anti-symmetric (A) mode numbers are numbered as, A0 fundamental mode, and A1…A3 for higher modes.

By extracting roots over a wide frequency range, dispersion curves of both symmetric (S) and anti-symmteric (A) Lamb waves can be studied. With the material properties given above, dispersion curves have been calculated up to the 2nd higher mode (A2 and S2), see Figure 4. A practicable technique when calculating dispersion curves from Equation 1 is to find the lowest frequency root of each mode first, and then use that root as a starting value. Some kind of minimization technique has to be used to find the exact root iteratively, keeping the frequency fixed and varying the wave number in small increments. When the lowest frequency root (first resonant wave number) at one frequency has been found with sufficient accuracy this wave number can be used as a starting value for the next frequency. This procedure is then repeated over the frequency range of interest.

At the lowest frequencies (here <3 kHz) only the two fundamental modes A0 and S0 exists. In this frequency range the S0 dispersion curve approaches the quasi-longitudinal wave velocity, which is the P-wave velocity along the plate. The A0 dispersion curve approaches the bending wave velocity of the plate: 200 to 400 m/s at these low frequencies. At higher frequencies A0 and S0 approach the Rayleigh wave velocity of the plate. It can be shown analytically that Equation 1 reduces to the Rayleigh wave dispersion equation in a homogeneous half space when the frequency approaches infinity. Practically Rayleigh wave motion develops at about 10 kHz in this example; this is where A0 and S0 merge together at the Rayleigh wave velocity. At these high frequencies and small wavelengths the finite-thickness plate appears as a semi-infinite medium and wave propagation is limited to the regions closest to the surfaces. Higher modes develop at their respective cut-off frequency, which is related to the thickness of the plate (Graff, 1975). At higher frequencies all higher modes approach the shear wave velocity of the plate. It should also be noted that all symmetric modes have an almost straight part where the phase velocity is close to the quasi-longitudinal wave velocity. All these different types of waves and velocities are indicated on the same dispersion curves in Figure 5. As mentioned earlier, the axis on the dispersion curves can be normalized with respect to the plate properties. In Figure 5 the frequency axis is multiplied with the plate thickness (f*d) and the phase velocity axis is divided by the shear wave velocity of the plate (c/V_S).

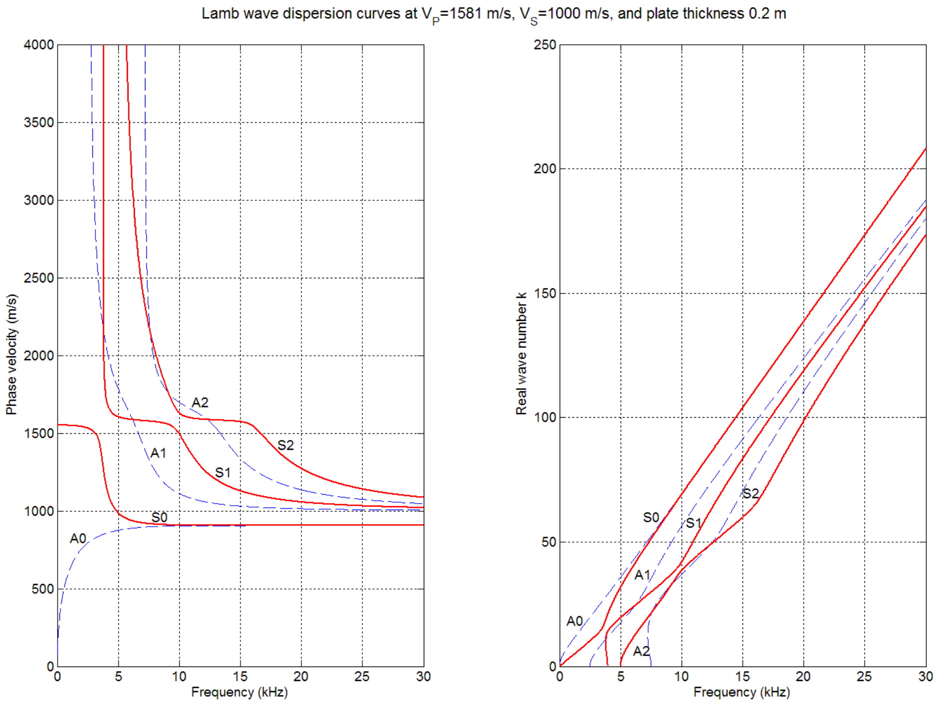


Figure 4. Lamb wave dispersion curves for a free plate with V_P=1581 m/s, V_S=1000 m/s (ν=0.167), and a thickness of 0.2 m. In the frequency-phase velocity domain (left) mode numbers increase upward with higher phase velocities. In the frequency-wave number domain (right) mode number increase downwards with lower wave numbers (compare with Figure 2).

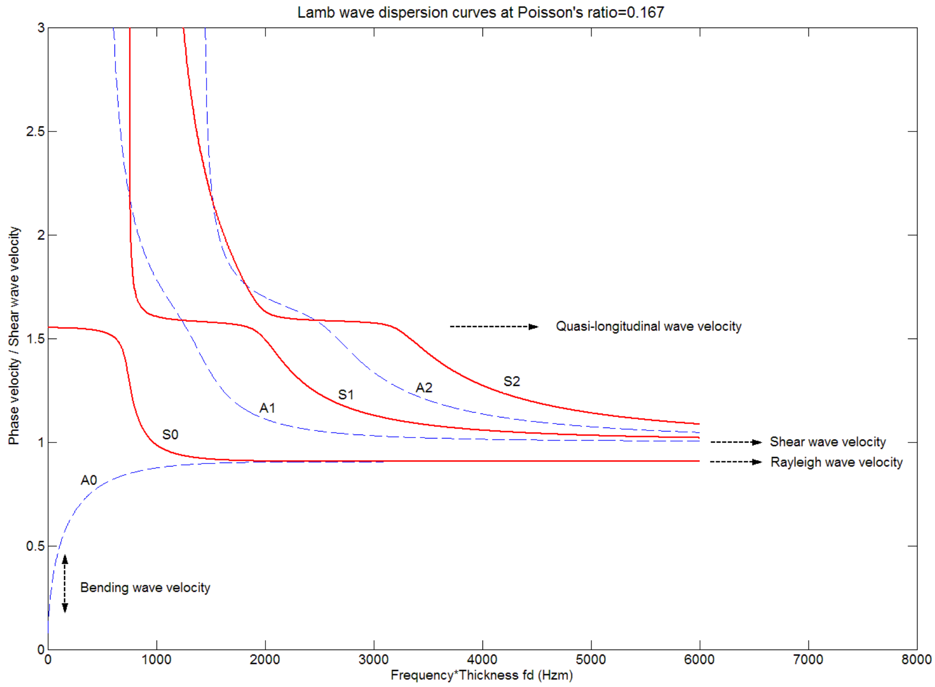


Figure 5. Lamb wave dispersion curves at a Poisson's ratio of 0.167. Axes are normalized with respect to the shear wave velocity and thickness of the plate. The different types of wave propagation are indicated with arrows.

Normalizing the axis with respect to the plate properties (as in Figure 5) significantly reduces computations when experimental dispersion curves are to be matched with theoretical dispersion curves. One only needs to calculate one set of dispersion curves at different Poisson's ratios. These dispersion curves can then be used as a database during the matching procedure. Normalized dispersion curves at fixed Poisson's ratios have been calculated by the authors and can be requested from nils.ryden@tg.lth.se. Very small increments of Poisson's ratio are usually not necessary because the dispersion curves are not very sensitive to changes in Poisson's ratio, which will be illustrated next.

The dispersion equation (Equation 1) is highly non-linear with respect to the plate properties. This implies that different portions of the dispersion curves are not equally sensitive to the plate properties. In Figure 6a-d the effect of a 10% increase in shear wave velocity (Figure 6a), Poisson's ratio (Figure 6b), E-modulus (Figure 6c), and thickness (Figure 6d) are displayed as dotted dispersion curves. The solid reference dispersion curves are the same as those used before. It should be noted that the low frequency range of A0 is not very sensitive to any of the plate properties. Still, this is the mode that has been used most widely in studies on seismic NDT of concrete plate structures and pavements. It is clear from Figure 6 that utilizing higher modes of lamb waves can increase the resolution of the results in terms of thickness and shear wave velocity or E-modulus. It can also be concluded that the Lamb wave dispersion curves are relatively insensitive to changes in Poisson's ratio. Actually there is always a frequency at each dispersion curve that is not affected by Poisson's ratio at all, termed Lamé modes (Graff, 1975). For the comparison on the sensitivity to changes in Poisson's ratio it should, however, be noted that the effect increases slightly with higher Poisson's ratios (here only 0.167).

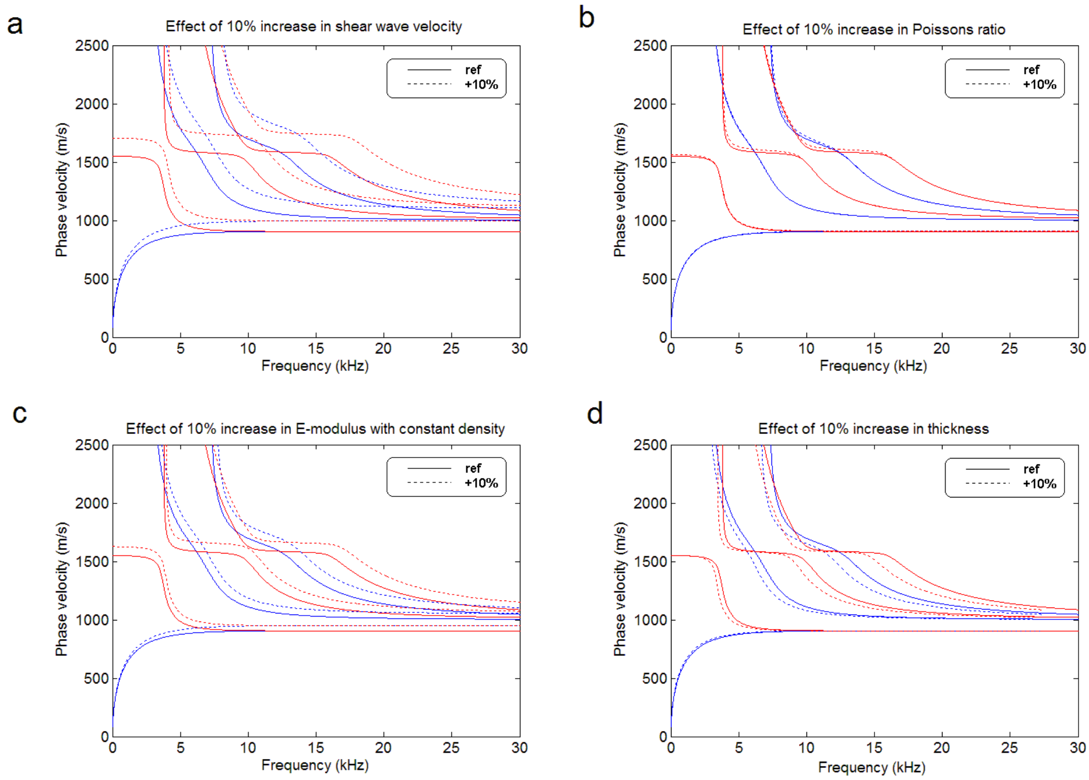


Figure 6. Parametric study on the sensitivity of multimodal Lamb wave dispersion curves on plate properties.

Field method

With a simple and cost effective field procedure and by utilizing the Multichannel Analysis of Surface Waves (MASW) processing technique (Park et al., 1998; Park et al., 1999) experimental Lamb wave dispersion curves can be measured. The multimodal nature of Lamb wave propagation implies that a transient source at the surface will easily generate multiple modes of propagation. Measured time histories along the surface will as a result be composed of a number of superposed modes. Therefore it is important that different modes of Lamb wave propagation can be delineate with a practical and cost effective approach.

The Multichannel Simulation with One Receiver (MSOR) data acquisition technique is used to record data. In the MSOR data acquisition method a multichannel record is obtained with only one receiver. It is fixed at a surface point and receives signals from several hammer impacts at incremental offsets (Ryden et al., 2001). All recorded signals are then compiled to make an equivalent multichannel record for dispersion analysis. The Portable Seismic Acquisition System (PSAS) is used to collect this data (Ryden et al., 2002a). With this system the signal from each impact is automatically streamed to the hard drive of a portable computer and data can be collected with only fractions of a second between the impacts. A schematic description of the field set-up is shown in Figure 7.

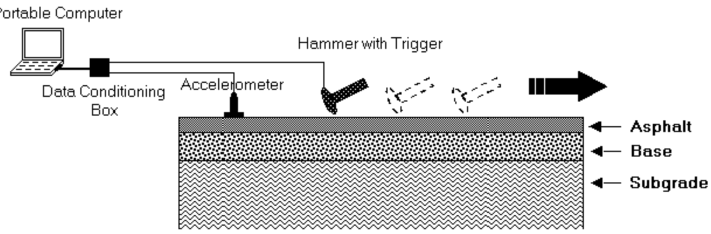


Figure 7. Schematic description of the MSOR measurement set-up.

The resulting multichannel record is automatically and objectively transformed to the frequency-phase velocity domain with the MASW processing technique (Park et al., 1998). With this technique it is possible to extract multimodal dispersion curves from a multichannel (or multichannel equivalent) data set. All coherent phase velocity patterns are mapped with respect to their relative energy level. Therefore the result (phase velocity image) shows how the total seismic energy is distributed between different frequencies and phase velocities. By applying this processing technique on synthetic data (from a finite difference model), multimodal Lamb wave dispersion curves have been observed (Ryden et al., 2002b).

Finally the measured experimental Lamb wave dispersion curves are matched to theoretical dispersion curves. By changing V_S, d, and ν, theoretical Lamb wave dispersion curves are matched manually or automatically to the high frequency dispersion curves in the phase velocity image. This whole procedure can be done in the field in a couple of minutes, provided that normalized dispersion curves have been calculated in advance as described in the previous section. The complete analysis scheme is exemplified next with a case study.

Case study—concrete bridge

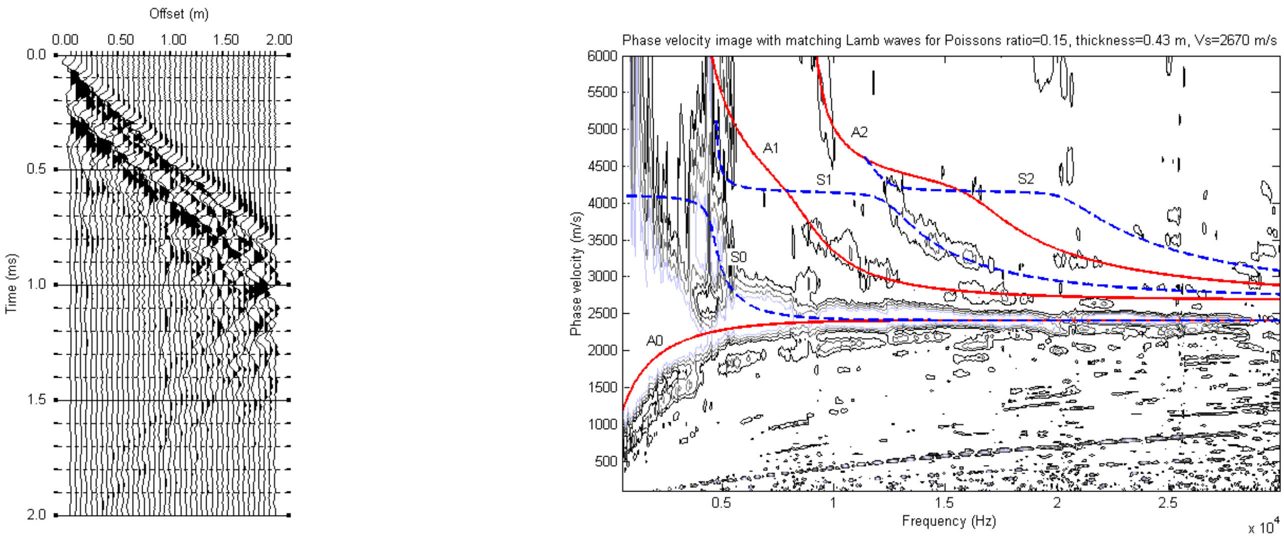
Data was collected at an old concrete railroad bridge in Malmoe, Sweden. The investigated concrete bridge was tested along the vertical walls on the south and north supports. This data set has been presented earlier in Ryden et al. (2002c). Along with the non-destructive seismic test, core samples from the same locations were also taken.

Following the MSOR method one accelerometer was located at zero offset (distance). While keeping the accelerometer at zero offset and by changing the impact points of the hammer from offset 0.05 m to 2.00 m with 0.05 m impact separation, data were collected with the PSAS system. A small (0.22 kg) carpenter hammer was used as source. A steel spike was used as a source coupling device to minimize source statics, possible operator related offset errors, and to maximize the bandwidth of the generated stress waves. At each offset 5 impacts were stacked with the spike kept in a fixed position. After the last impact all individual traces are saved in a multichannel format as presented in Figure 8a.

To evaluate phase velocities at each frequency, the time record has been automatically transformed to the frequency-phase velocity domain by using the MASW phase velocity analysis scheme (Park et al., 1998). Several energy crests, which represent different modes of Lamb wave propagation, are visible in the contour plot (Figure 8b).

In Figure 8b the measured data is compared with theoretical Lamb wave dispersion curves corresponding to a 0.43 m thick free plate with a shear wave velocity of 2670 m/s and a Poisson's ratio of 0.15. In this example only the fundamental modes of symmetric and anti-symmetric Lamb waves have been fully resolved in the measurements. The higher modes S1, S2, A1, and A2 show only week energy in limited parts of the spectrum. Since the higher modes are more sensitive to changes in the material properties (see Figure 6) even these limited parts of the spectrum are useful when matching the measured dispersion curves with the theoretical dispersion curves. The density (ρ), 2400 kg/m3, has no effect on the dispersion curves of pure Lamb waves and is only used to calculate the E-modulus from V_S and ν. The resulting dynamic E-modulus of the concrete support is calculated to 39.4 Gpa (Table 1) from Equation 4.

$$E = 2\rho V_S^2 (1 + \nu) \qquad \text{Eq. 4}$$



(a) (b)

Figure 8. Result from the seismic field test at the north side of the investigated concrete bridge. In (a) the collected raw data is presented in a multichannel format, (b) shows the corresponding data in the frequency phase velocity domain, as it appears after the MASW transformation method.

The presented result above was collected at the north support of the concrete bridge. The south support was also tested in a similar manner. Alongside the non-destructive seismic tests, core drillings were also conducted at the same locations. The cores were tested with the free-free resonant column method (Richart et al., 1970). A commercially available system, *Grindosonic*, was used for this test. In the free-free resonant test the seismic shear and compression wave velocities are indirectly evaluated from the longitudinal and torsional resonance frequencies of the sample. The resulting E and ν from this laboratory test are presented in Table 1 together with the results from the non-destructive field test.

A comparison of the results shows a slightly higher modulus from the seismic MSOR field test. The discrepancy between the results could to some extent be related to the approximate geometrical correction factors used in the calculation of Young's modulus from the measured resonant frequencies on the drilled cores. In this study the ASTM C1259-94 standard was used to calculate the presented material properties from the free-free resonant test. In Table 1 E and ν are presented both with and without the geometrical correction factor. In the field test pure Lamb wave dispersion curves from a free plate has been used to match the measured dispersion curves. In reality one side of the concrete wall is in contact with soil fill and as mentioned earlier this coupling is not accounted for. Another factor affecting the different methods is the difference in test volume between the two tests.

Table 1. Evaluated dynamic stiffness properties of the tested concrete.

Location	Destructive Grindosonic ASTM correction factors		Destructive Grindosonic No correction factors		Non-destructive MSOR field test	
	E (GPa)	ν	E (GPa)	ν	E (GPa)	ν
South side	40.7	0.18	42.9	0.18	43.6	0.20
North side	37.3	0.15	39.9	0.15	39.4	0.15

Discussion

It is important to remember that pure Lamb wave dispersion curves are only valid strictly for an elastic isotropic plate in vacuum. In this pure case no energy is leaking into the surrounding medium. However, provided that the velocity contrast is large, like soil-concrete, it has been shown by several researchers that Lamb wave dispersion curves are practically not affected by the interface medium (Jones and Thrower, 1965; Martincek, 1994). Based on experimental tests Martincek (1994) concluded that normalized frequencies higher than a *fd* product of 0.15 times the phase velocity ($fd > 0.15c$) are practically not affected by coupling to a lower layer. But, nevertheless, the geometry of the structure should be as close to a free plate as possible. For more complex structures of irregular shape this simplified inversion technique is not possible.

The usefulness of the presented method is related to the simple testing procedure and analysis. Testing in itself is a simple procedure. Approximately 10 minutes were spent on the data collection at 40 different offsets from the receiver, and with the laptop at site the evaluation can be performed immediately. The calculated result can be seen as a mean value for the property of the concrete volume affected by the measured seismic waves. This gives more trustworthy information than the point estimates of the property gained by core drilling. It is also important to remember that the calculated response actually is a mean value based on every hammer blow that is measured. This implies that a standard deviation of the evaluated properties also can be estimated. Since reliability analysis is becoming a more and more common tool for assessment, the variability of the concrete is important. A smaller variability can be used to increase the safety of the structure.

Evaluated dynamic stiffness properties of the concrete can be reduced to static values with known empirical relations (Nagy, 1997). From the static E-modulus it is also possible to estimate the compression strength.

Conclusions

With simple field measurements using one receiver and one source (MSOR) and by utilizing the MASW data processing method, multimodal Lamb wave testing can be performed directly in the field. Measuring shear and compression wave velocity in concrete is common for non-destructive test methods. The unique features that makes this method highly efficient is the simple and cost effective data acquisition and the ability to extract multimode dispersion curves in a robust and objective manner. The case study presented shows good agreement with data from more expensive tests based on core drillings.

In this paper it has also been demonstrated how pure Lamb wave dispersion curves are calculated from the dispersion equation. This can be a tricky task and is therefore described in detail. However dispersion curves for a given Poisson's ratio only need to be calculated once. By normalizing the axis of the dispersion curves with respect to the shear wave velocity and the thickness of the plate, calculated dispersion curves can be used for any plate with any shear wave velocity and any thickness. Normalized dispersion curves at fixed Poisson's ratios have been calculated by the authors and can be requested from nils.ryden@tg.lth.se.

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